STUDY CONCERNING THE LAMINAR SUBLAYER REGION OF A TURBULENT BOUNDARY LAYER WITH INJECTION

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Results are shown of an experimental study concerning the laminar sublayer region of a turbulent boundary layer with injection.

A turbulent boundary layer at a porous plate has been the subject of numerous studies, inasmuch as such a type of flow is encountered in many practical problems. It is well known, for example, that both the friction and the thermal flux at the wall decrease considerably even when the rate of injection is very low.

At the present time there is already a sufficient volume of test data available pertaining to the mean-velocity profiles in the fully developed turbulent region of a boundary layer with injection [1-5]. Much scarcer is the information available about the pulsating components in this region, and almost no measurements have been made in the viscous sublayer.

In this study the authors have attempted to examine the flow pattern in the viscous sublayer.

The tracing method, which had been perfected by the authors of [6], was employed here too. The gist of this method was to photograph light-reflecting fine particles implanted into the stream and illuminated laterally from a pulsating light source.

The test apparatus is shown schematically in Fig. 1. The test segment was made up of a rectangular channel 30 × 30 mm in cross section with translucent top and side walls.

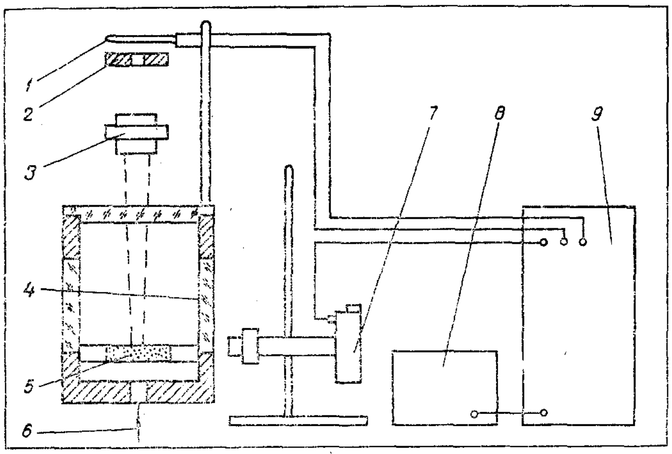

Fig. 1. Basic schematic diagram of the test instrumentation: (1) lamp, (2) slitted screen, (3) camera objective, (4) translucent channel walls, (5) porous plate, (6) injection, (7) camera, (8) audio generator, (9) electronic stroboscope.

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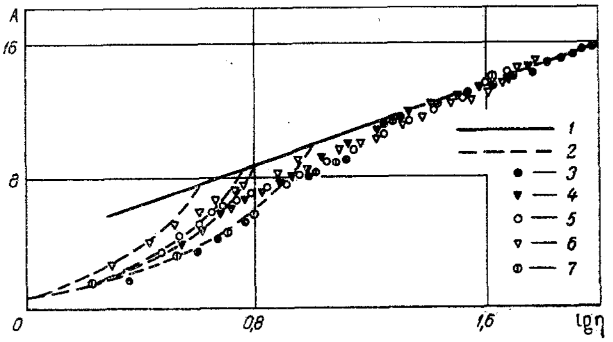


Fig. 2. Distribution of mean velocity near the wall: Eq. (1) and A = 2.5 ln η + 4.9 (1), Eq. (3) and A = $(\exp(\eta w_b/2u^*) - 1)\, 2u^*/w_b$ (2), b = 0 and Re** = 900 (3), b = 2.54 and Re** = 970 (4), b = 3 and Re** = 1240 (5), b = 3.8 and Re** = 1320, $A \equiv 2u^*/w_b [((w_b/u^*)\varphi + 1)^{1/2} - 1]$ (6), according to tests by H. Reihardt [7] (7).

Into the bottom wall of the channel was mounted a porous 6 × 200 mm plate. The tracks were photographed at a section 230 mm away from the channel entrance. The active medium was distilled water at room temperature. The size of particles implanted into the stream was in the 5-20 μm range.

The optical instrumentation is also shown in Fig. 1. A slit directly in front of the IFK-120 flashing lamp was projected by means of a wide-aperture objective and through the translucent channel lid into the test zone of the stream. On the same test zone was focused the camera with an attachment for ×8 magnification. With a mirror-type view finder it was easy to orient the camera toward the illuminated stream zone near a channel wall and to track the flow during picture taking. A series of flashes of the lamp was triggered by an electronic stroboscope to produce on the photographic film a series of images of the same particle (intermittent tracking). The development of many frames contributed to a reliable determination of the longitudinal $\bar{u}$-components and the vertical $\bar{v}$-components of mean flow velocities as well as the rms of their pulsations $\sqrt{u'^2}$ and $\sqrt{v'^2}$.

First we measured by this method the velocity and the turbulence level in the main stream. The turbulence level here was 3.4%. The velocity in the stream outside the boundary layer was measured by this method and also with a Pitot tube: the results did not differ by more than 0.5%. The velocity distribution in the viscous sublayer without injection agreed with the results in [7].

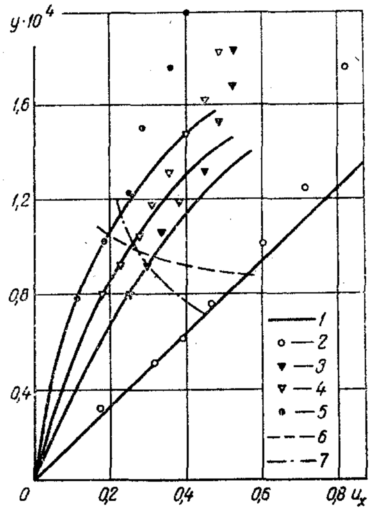
The velocity distributions in the boundary layer at the porous plate, without and with injection, are shown in Fig. 2. The straight line represents the "wall law" based on studies by W. H. Dorrance and F. J. Dore [8] and expressed as

$$\frac{2u^*}{w_b}\left[\left(1+\frac{w_b\varphi}{u^*}\right)^{1/2}-1\right]=\frac{1}{k}\ln\eta+c, \tag{1}$$

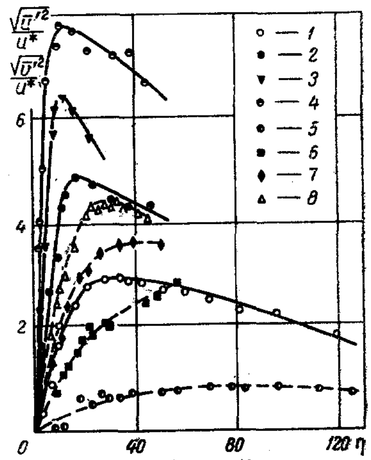
where k is an empirical constant whose value has been assumed to correspond to a flow without injection (k = 0.4). Constant c is, generally, a function of the injection rate, as had already been noted by M. W. Rubesin [9], W. H. Dorrance and F. J. Dore [8], and T. N. Stevenson [1, 2, 10]. The measurements made by T. N. Stevenson [10], H. Micklay and R. S. Davis [3], V. K. Johnson and C. J. Scott [4] have shown, as S. Kinney has noted in [11], that in a boundary layer subject to the "wall law" the constant c

TABLE 1. Initial Data and Calculated Parameters

b	w_0, m/sec	$w_b \cdot 10^2$, m/sec	Re**	δ,** mm	$u^* \cdot 10^2$, m/sec	$\nu \cdot 10^6$, m²/sec	ξ_1^*	ξ_1
0	1,86	0	900	0,55	8,8	1,15	0,077	0,0116
2,54	1,75	1	970	0,55	4,5	1,0	0,063	0,0155
3,0	1,82	1,16	1240	0,68	3,78	1,0	0,05	0,0174
3,8	1,95	1,56	1320	0,68	2,7	1,0	0,066	0,021



(Fig. 3)



(Fig. 4)

Fig. 3. Distribution of mean velocity in the viscous sublayer: Eq. (2) (1), b = 0 and Re** = 900 (2), b = 2.54 and Re** = 970 (3), b = 3 and Re** = 1240 (4), b = 3.8 and Re** = 1320 (5), $Re_1 = u_1 y_1 / \nu$ = const (6), $Re_1^* = u_1^* y_1/\nu$ = const. Dimension y_1 (m), u_x (m /sec).

Fig. 4. Dimensionless profiles of the rate of longitudinal and transverse velocity pulsations: $\sqrt{\overline{u'^2}}/u^*$ with b = 0 (1), 2.54 (2), 3 (3), 3.8 (4); $\sqrt{\overline{v'^2}}/u^*$ with b = 0 (5), 2.54 (6), 3 (7), 3.8 (8).

has almost the same value 4.9 over the entire range of injection velocity within which it is still possible to measure friction at the wall with sufficient accuracy.

The velocity distribution in the usually assumed thin (viscous) sublayer of a turbulent boundary layer with injection is described by the equation:

$$\varphi = \frac{u^*}{w_b}\left(\exp\frac{\eta w_b}{u^*} - 1\right), \tag{2}$$

which can be transformed into

$$\frac{2u^*}{w_b}\left[\left(1+\frac{w_b\varphi}{u^*}\right)^{1/2} - 1\right] = \left(\exp\frac{\eta w_b}{2u^*} - 1\right)\frac{2u^*}{w_b}. \tag{3}$$

Here the left-hand sides of Eqs. (1) and (3) are identical.

The main error in the test data shown in Fig. 2 was due to the inaccurate determination of local dynamic velocities. When $w_b = 0$, the local skin-friction coefficient in the expression for the dynamic velocity was found from the velocity profile near the wall and also by the Karman formula

$$C_{f_0} = \frac{2}{(2{,}5\ln Re^{**} + 3{,}8)^2}.$$

The values obtained for the dynamic velocity by these two methods respectively did not differ by more than 1.5%. The value of τ_b with injection was found from the measured velocity distribution in the viscous sublayer, under the assumption that this distribution could be described by Eq. (2).

It is to be noted that this value of τ_b corresponds to the value of τ_b found by the extremal Kutateladze–Leont'ev equation [12]:

$$\Psi = \left(1 - \frac{b}{b_{cr}}\right)^2 \tag{4}$$

with $b_{cr} = 5.5$, according to [13].

Assuming Eq. (2) to be valid for a viscous sublayer, we could estimate the boundary of this sublayer η_1, (Fig. 3) from the deviation of test points from this curve. For a boundary layer without injection, such estimates yielded values for η_1 which agreed with the Hudimoto model [14], where $\eta_1 \sim 6.1$ (at $\chi = 0.4$ and

$c = 4.9$). The relative variation of the dimensionless velocity ω_1 along this boundary of the viscous sublayer with injection $\omega_1/\omega_{10} = f(b)$ ($\omega_{10} = (u_1/u_0)_0$ denoting the velocity without injection) agreed with the results based on the two-layer model [15, 16]:

$$\frac{\omega_1'}{\omega_{10}'} = f(b).$$

These values of the dimensionless velocities ω_1', ω_{10}' could be easily obtained by solving simultaneously Eqs. (1) and (3). The absolute values ω_1, ω_1', of course, differed considerably.

There are several known approaches to establishing the stability criteria for a viscous sublayer. For instance, M. W. Rubesin [9] has suggested a constant Reynolds number on the basis of the velocity at the sublayer boundary ($Re_1 = u_1 y_1/\nu =$ const.), while E. R. van Driest [17] has suggested a constant Reynolds number on the basis of the dynamic velocity at the sublayer boundary ($Re_1^* = u_1^* y_1/\nu =$ const.). Inasmuch as the definition of y_1 is tentative, our test data do not favor any of these criteria. It can only be noted that the curves in Fig. 3 which correspond to $Re_1 =$ const. and $Re_1^* =$ const. are close.

The values obtained for ω_1 (with $Re =$ const. or $Re_1^* =$ const.) allow us to estimate the effect which the finiteness of the Re^{**} number has on Eq. (4). An analysis has shown this effect to be appreciable enough when the finiteness of Re^{**} is considered in the determination of b_{cr} [12].

For determining the upper boundary of the transition region η_1^* we used the data which had been obtained on the distribution of longitudinal velocity pulsations. As is well known, the generation and the dissipation of kinetic energy of turbulence near the impermeable wall of a transition layer pass through a distinct maximum. An analogous pattern is observed also with injection. In Fig. 4 is shown $\sqrt{\overline{u'^2}}/u^*$ as a function of η and b. If one assumes the coordinate at which $\sqrt{\overline{u'^2}}/u^*$ reaches its maximum to be the upper boundary of the transition layer η_1^*, as I. J. Shigemitsu has assumed in [18], then one may conclude that the thickness of the transition layer η_1^* decreases as the rate of injection increases. The quantity $\xi_1^* = y_1^*/\delta$ appears to be more conservative, although it also decreases (except when $b = 3.8$, see the data in Table 1). The quantity ξ_1, on the other hand, increases with the injection rate. Thus, the transition zone as a fraction of the total boundary-layer thickness decreases as the injection rate increases. It ought to be noted here that, as the parameter b decreases, the peak of longitudinal pulsations become sharper.

We also obtained the distribution of longitudinal and transverse velocity pulsations, from which we could determine the variation in state pressure across the boundary layer. The static pressure in a given section of a boundary layer is

$$p = p_0 - \rho\overline{v'^2}.$$

The variation of static pressure across the boundary layer, due to the $\rho\overline{v'^2}$ component, did not exceed 1% of the dynamic head $\rho u_0^2/2$ in all our tests.

It must be noted that this method of tracing the stream with particles is more difficult to apply at higher injection rates, because then the particles move away from the channel walls.

NOTATION

u', v'	are the longitudinal and transverse components of pulsating velocity;
$u^* = \sqrt{\tau_b/\rho}$	is the dynamic velocity, respectively;
$\varphi = u_x/u^*$	is the dimensionless velocity;
$\eta = u^* y/\nu$	is the dimensionless distance from the wall;
Cf_0	is the friction coefficient at a flat impermeable plate in an infinitely large isothermal stream;
$Re^{**} = u_0\delta^{**}/\nu$	is the characteristic Reynolds number for a boundary layer;
$b = 2\rho_b w_b/\rho_0 w_0 Cf_0$	is the wall permeability;
$\Psi = (C_f/C_{f_0})Re^{**}$	is the relative change in the friction coefficient at $Re^{**} =$ idem;
p	is the static pressure;
ρ	is the density of the medium;
δ^{**}	is the momentum thickness;
w	is the velocity;
$\omega = w/w_0$	is the dimensionless velocity;
ξ_1^*, η_1^*	is the upper boundary of the transition zone.

Subscripts

b, 0 refer to the wall and to the outer boundary of the boundary layer respectively;
cr refers to the stagnation point;
1 refers to the viscous-sublayer boundary.

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